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NATURAL CONVECTION THROUGH RECTANGULAR OPENINGS IN PARTITIONS—2

HORIZONTAL PARTITIONS*

W. G. BROWN
Building Services Section, Division of Building Research,
National Research Council, Ottawa, Canada

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Abstract—Natural convection through square openings in a horizontal partition for the case of heavier fluid above the partition is investigated using air as the fluid medium. The test results relating the Nusselt number to the Grashof number and to the ratio of opening thickness to opening width are found to agree generally with the requirements of theory. Although the Prandtl number for air remained constant in all tests, it was nevertheless possible, with the help of the theory, to show the approximate influence to be expected for any value of the Prandtl number. Because of the high thermal resistance of the partition material the test results may also be expected to apply directly to mass transfer. The following range of variables was covered in the tests: Grashof number Gr_H based on partition thickness and air temperature difference across the opening, $3 \times 10^4 < Gr_H < 4 \times 10^7$; ratio of partition thickness to the side of the square opening H/L, $0{\cdot}0825 < H/L < 0{\cdot}66$, with openings of 6×6 in, 9×9 in and 12×12 in.

IN A previous paper (Part 1), the theory and experimental results for natural convection across openings in vertical partitions were given. In this situation buoyancy forces resulting from temperature or concentration differences between the fluids on either side of a vertical partition cause a fluid interchange across a partition opening with resultant heat or mass transport.

To complete the investigation of natural convection across openings in partitions the theory and experimental results for horizontal partitions will now be presented. As far as is known no previous work of this kind has been carried out. The situation of greatest importance is that where the fluid above the opening in a partition has the greater density; an unstable condition then arises, and an interchange of lighter and heavier fluid takes place. A rather surprising result of this interchange, which would not be immediately recognized but which is predicted by the theory and was verified experimentally, is that heat- or mass-transfer rates increase with increasing partition thickness.

The following theoretical considerations are carried out in considerable detail in order to introduce a method of analysis suited to convection problems generally. The need for a systematic approach to such problems arises, in particular, when several dimensionless variables are to be related. For this condition it is often difficult, if not impossible, to obtain a relationship among the variables by experimental means alone that is not partly or entirely empirical. In experimental work an attempt is often made to relate the dimensionless variables of the problem as products of powers, but since the range of variation of one or more of the variables is usually limited, no complete equation can be written for the phenomenon. Moreover, the exponents on the dimensionless variables themselves are often inter-dependent; consequently even small experimental errors may cause some of the exponents to have apparent values that a theoretical investigation could have shown to be impossible.

* For Nomenclature, see Part I.

THEORY

In the situation shown in Fig. 1, two sealed cavities containing fluid at densities ρ_1 and ρ_2 ($\rho_1 > \rho_2$), with temperatures T_1 and T_2 and concentrations c_1 and c_2, are separated by a partition of thickness H having an opening of characteristic width L (length of a side for a square opening). The partition is assumed to be impermeable to heat or mass transfer.

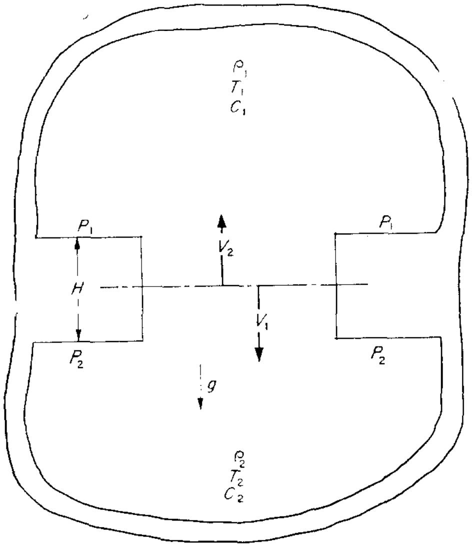

FIG. 1. Schematic representation of natural convection through an opening in a horizontal partition.

Since the condition of the fluid at the opening is inherently unstable, no steady distribution of flow can be assumed. For determining the general relationship between variables, however, any flow distribution in which the lighter fluid flows upward with velocity V_2 and the heavier fluid flows downward with velocity V_1 may be considered. The pressures p_1 and p_2 at the level of the top and bottom of the partition are presumed everywhere constant in the horizontal plane.

Neglecting any interchange of fluid in the horizontal direction, Bernoulli's equation yields, for the lighter fluid flowing upward

$$p_2 - p_1 = \rho_2 \frac{V_2^2}{2} + l_2 + \rho_2 g H \qquad (1)$$

and for the heavier fluid flowing downward

$$p_2 - p_1 = -\rho_1 \frac{V_1^2}{2} - l_1 + \rho_1 g H \qquad (2)$$

or, combining equations (1) and (2),

$$(\rho_1 - \rho_2) g H = \rho_1 \frac{V_1^2}{2} + \rho_2 \frac{V_2^2}{2} + (l_1 + l_2). \qquad (3)$$

Here g is the acceleration due to gravity and l_1 and l_2 are pressure losses due to entrance into the opening and fluid friction.

The condition of no net flow across the opening requires that

$$\rho_1 E_1 V_1 = \rho_2 E_2 V_2 \qquad (4)$$

where E_1 and E_2 are the cross-sectional areas over which the flow occurs.

If ρ_1 and ρ_2 do not differ greatly it is reasonable to assume that both V_1 and V_2 and consequently E_1 and E_2 are approximately equal. With the further assumption that $l_1 \simeq l_2$, equation (3) then becomes:

$$\left(\frac{\rho_1 - \rho_2}{\bar{\rho}}\right) gh = \frac{\Delta\rho}{\bar{\rho}} gH = V^2 + \frac{2l}{\bar{\rho}}. \qquad (5)^*$$

From knowledge of the general behaviour of fluid flow in pipes and conduits it can be assumed that for small ranges of all variables

$$l = C\bar{\rho}\frac{V^2}{2}\left(\frac{H}{L}\right)^a\left(\frac{VL}{\nu}\right)^b \qquad (6)$$

where C is a constant and ν is the kinematic viscosity. The exponent a must lie between 0 and 1 and exponent b must lie between 0 and -1. (Provided only that the flow is not transitional, i.e. as occurs in a pipe at the critical value of Re.)

* In a study of natural convection in an insulated vertical tube with the higher temperature at the lower end, Grassmann [1] takes a similar approach in deriving the flow equations. Owing to the large ratio of height to opening size (pipe diameter), however, his assumptions for the pressure losses do not apply to the present problem. Similarly, the assumption that all heat transfer takes place in the lateral direction between the two streams of fluid is not valid in the present case.

Insertion of equation (6) into equation (5) gives

$$\frac{\Delta\rho}{\bar{\rho}} gH = V^2\left[1 + C\left(\frac{H}{L}\right)^a\left(\frac{VL}{\nu}\right)^b\right]. \quad (7)$$

Again for a small range of $C\ (H/L)^a\ (VL/\nu)^b$, equation (7) can be approximated by

$$\frac{\Delta\rho}{\bar{\rho}} gH = V^2 C_1\left[\left(\frac{H}{L}\right)^a\left(\frac{VL}{\nu}\right)^b\right]^d \quad (8)$$

where d must now lie between 0 and 1. C_1 is another constant. From equation (8)

$$V^{2+bd} = \frac{\Delta\rho}{\bar{\rho}} gH/C_1\left(\frac{H}{L}\right)^{ad}\left(\frac{L}{\nu}\right)^{bd} \quad (9)$$

or

$$V = \frac{C_2\left(\frac{\Delta\rho}{\bar{\rho}} gH\right)^{1/(2+bd)}}{\left(\frac{H}{L}\right)^{ad/(2+bd)}\left(\frac{L}{\nu}\right)^{bd/(2+bd)}} \quad (10)$$

where the exponent bd lies now between 0 and -1 and ad lies between 0 and 1, with C_2 as a new constant.

Having obtained an expression for velocity, the heat- and mass-transfer rates across the opening become respectively

$$\dot{q} = \bar{\rho} c_p \frac{L^2}{2}(T_2 - T_1)V \quad (11)$$

and

$$\dot{m} = \bar{\rho}\frac{L^2}{2}(c_2 - c_1)V \quad (12)$$

with c_p as the specific heat.

Introducing the heat-transfer coefficient h_T and the mass-transfer coefficient h_m, defined as

$$h_T = \dot{q}/L^2(T_2 - T_1)$$

and

$$h_m = \dot{m}/L^2\bar{\rho}(c_2 - c_1),$$

equations (10–12) lead to the following equations in dimensionless form: for heat transfer

$$\frac{h_T H}{k} = Nu_H = \frac{C_2}{2}\left(\frac{\Delta\rho g H^3}{\bar{\rho}\nu^2}\right)^{1/(2+bd)}\left(\frac{L}{H}\right)^{(ad-bd)/(2+bd)}\left(\frac{c_p\bar{\rho}\nu}{k}\right) = C_3 Gr_H^{1/(2+bd)}\left(\frac{L}{H}\right)^{(ad-bd)/(2+bd)} Pr \quad (13)$$

and for mass transfer

$$\frac{h_m H}{D} = Sh_H = C_3 Gr_H^{1/(2+bd)}\left(\frac{L}{H}\right)^{(ad-bd)/(2+bd)}\left(\frac{\nu}{D}\right) = C_3 Gr_H^{1/(2+bd)}\left(\frac{L}{H}\right)^{(ad-bd)/(2+bd)} Sc. \quad (14)$$

Here k is the thermal conductivity of the fluid and D is the diffusion coefficient.

Nu_H = Nusselt number based on partition thickness,
Sh_H = Sherwood number based on partition thickness,
Gr_H = Grashof number based on partition thickness,
Pr = Prandtl number,
Sc = Schmidt number.

By summing the exponents on H in equation (13) or (14) it is readily found that either h_T or h_m is proportional to $H^{(1-ad)/(2+bd)}$ and because of the limits on ad and bd then $(1 - ad)/(2 + bd)$ lies between 0 and 1. Thus the heat or mass transfer will either remain constant or increase with increasing partition thickness. Equations (13) and (14) can also be used as a partial check on experimental results because the permissible range for the exponent on one dimensionless group is conditioned by the exponent on the other. If, for example, bd is found to be $-0{\cdot}2$ then the exponent on (L/H) in equations (13) and (14) must lie between $(0 + 0{\cdot}2)/1{\cdot}8 = \frac{1}{9}$ and $(1 + 0{\cdot}2)/1{\cdot}8 = \frac{2}{3}$.

In employing equations (11) and (12) it was tacitly assumed that essentially no heat or mass transfer takes place by thermal conduction or mass diffusion in the fluid. For fluids with high thermal conductivity and diffusion coefficients additional consideration must be made.

Assuming negligible heat or mass transfer in the horizontal direction, the equations for the heat and mass conservation in the opening are

$$V\frac{\mathrm{d}T}{\mathrm{d}z} = \alpha\frac{\mathrm{d}^2T}{\mathrm{d}z^2} \qquad (15)$$

and

$$V\frac{\mathrm{d}c}{\mathrm{d}z} = D\frac{\mathrm{d}^2c}{\mathrm{d}z^2} \qquad (16)$$

where $\alpha(= k/\rho c_p)$ is the thermal diffusivity and z is the distance in the vertical direction. Integration of these equations gives the expressions

$$T - T_1 = (T_2 - T_1)\left[\frac{\exp(Vz/\alpha) - 1}{\exp(VH/\alpha) - 1}\right] \qquad (17)$$

(for temperature), and

$$c - c_1 = (c_2 - c_1)\left[\frac{\exp(Vz/D) - 1}{\exp(VH/D) - 1}\right] \qquad (18)$$

(for concentration). It will be noted that equations (17) and (18) reduce to the pure conduction and diffusion forms

$$(T - T_1)/(T_2 - T_1) = z/H = (c - c_1)/(c_2 - c_1),$$

for zero velocity or large α and D; and to the pure convection forms $T = T_1$ (constant) and $c = c_1$ (constant) for high velocities or small α and D.

The heat and mass being transported across area $E_1 \simeq L^2/2$ are respectively

$$\dot{q}_E = \left(V\rho(i_0 + c_pT) - k\frac{\mathrm{d}T}{\mathrm{d}z}\right)L^2/2 \qquad (19)$$

and

$$\dot{m}_E = \left(V\rho c - \rho D\frac{\mathrm{d}c}{\mathrm{d}z}\right)L^2/2 \qquad (20)$$

where $(i_0 + c_pT)$ is the enthalpy of the fluid. After substituting for $\mathrm{d}T/\mathrm{d}z$ and $\mathrm{d}c/\mathrm{d}z$, obtained from equations (17) and (18), equations (19) and (20) become

$$\dot{q}_E = V\rho\frac{L^2}{2}\left\{i_0 + c_p\left[\frac{T_1\exp(VH/\alpha) - T_2}{\exp(VH/\alpha) - 1}\right]\right\} \qquad (21)$$

and

$$\dot{m}_E = V\rho\frac{L^2}{2}\left\{\frac{c_1\exp(VH/D) - c_2}{\exp(VH/D) - 1}\right\}. \qquad (22)$$

The net heat and mass transfer across the partition due to fluid flow in both directions is thus respectively,

$$\dot{q} = V\bar{\rho}c_p\frac{L^2}{2}\left\{\left[\frac{T_1\exp(VH/\alpha) - T_2}{\exp(VH/\alpha) - 1}\right] - \left[\frac{T_1\exp(-VH/\alpha) - T_2}{\exp(-VH/\alpha) - 1}\right]\right\}$$

$$= V\bar{\rho}c_p\frac{L^2}{2}(T_1 - T_2)\left[\frac{\exp(VH/\alpha) + 1}{\exp(VH/\alpha) - 1}\right] \qquad (23)$$

and

$$\dot{m} = V\bar{\rho}\frac{L^2}{2}(c_1 - c_2)\left[\frac{\exp(VH/D) + 1}{\exp(VH/D) - 1}\right]. \qquad (24)$$

It will be recognized that these equations reduce to the pure convection forms:

$$\dot{q} = V\bar{\rho}c_p\frac{L^2}{2}(T_1 - T_2) \qquad (25)$$

and

$$\dot{m} = V\bar{\rho}\frac{L^2}{2}(c_1 - c_2) \qquad (26)$$

for high velocity or low α or D. Similarly, for low velocity or high α and D the equations reduce to the pure conduction and diffusion forms:

$$\dot{q} = kL^2\frac{(T_1 - T_2)}{H} \qquad (27)$$

and

$$\dot{m} = \bar{\rho}DL^2\frac{(c_1 - c_2)}{H}. \qquad (28)$$

Equation (10) can now be inserted into equations (23) and (24) and the Nusselt and Sherwood numbers evaluated. It will be noted, however, that the density difference $\Delta\rho$, which was initially assumed to be equal to $\rho_1 - \rho_2$, must now be taken as an average value since both temperature and concentration, on which density depends, now vary throughout the height H.

Writing

$$\frac{\Delta\rho}{\rho} = \beta_T\Delta T + \beta_m\Delta c \qquad (29)$$

where β_T and β_m are the coefficients of thermal and mass expansion for the fluid, then the average of ΔT and Δc over the height H can be inserted to obtain the average $\Delta\rho$.

From equation (17) the average temperature difference over the height H between upward- and downward-flowing fluid is

$$\Delta T_{\text{avg}} = \frac{(T_2 - T_1)}{H} \int_0^H \left\{ \left[\frac{\exp(Vz/\alpha) - 1}{\exp(VH/\alpha) - 1}\right] - \left[\frac{\exp(-Vz/\alpha) - 1}{\exp(-VH/\alpha) - 1}\right] \right\} dz$$

$$= (T_1 - T_2)\left[\frac{\exp(VH/\alpha) + 1}{\exp(VH/\alpha) - 1} - \frac{2\alpha}{VH}\right]. \quad (30)$$

Similarly, from equation (18) the average concentration difference is

$$\Delta c_{\text{avg}} = (c_1 - c_2)\left[\frac{\exp(VH/D) + 1}{\exp(VH/D) - 1} - \frac{2D}{VH}\right]. \quad (31)$$

Equations (30) and (31) can now be inserted into equation (29), and this with equation (10) can be inserted into equations (23) and (24) to obtain the Nusselt and Sherwood numbers. Consequently, for the general case of all fluids the relationship between the Nusselt or Sherwood numbers and the remaining variables can be expressed as

$$Nu = f\,[Gr_H, Pr, H/L, \beta_T(T_2 - T_1), Sc] \quad (32)$$

and

$$Sh = f\,[Gr_H, Sc, H/L, \beta_m(c_2 - c_1), Pr] \quad (33)$$

where f signifies the same function in both cases.

Equations (32) and (33) are an interesting example, obtained directly from theory, in which heat and mass transfer are interrelated. The same kind of relationship would be obtained, of course, in the case of two-component mass transfer in place of heat transfer plus mass transfer. For the special case $Pr = Sc$, equations (32) and (33) reduce to

$$Nu = f_1(Gr_H, H/L, Pr) \quad (34)$$

and

$$Sh = f_1(Gr_H, H/L, Sc). \quad (35)$$

Application of the theory

The foregoing equations are approximate to a considerable degree. Nevertheless they can be used in conjunction with existing data from other flow problems to estimate the general magnitude of Nu and Sh, which would be expected in a practical situation. In conjunction with limited experimental results obtained with a given fluid, the equations may also be used to extrapolate data for other fluids. To illustrate these procedures, the conditions to be expected for air, the fluid used in the tests reported in the following section on experimental results, will be considered. Returning to equation (6), it is known from hydraulic experiments that the head loss l at a square entrance into a pipe can be expressed as $l = 0{\cdot}5\rho(V^2/2)$. Similarly, for a re-entrant pipe $l = 1{\cdot}0\rho(V^2/2)$.

Assuming that these values apply approximately for the situation in Fig. 1, equation (5) becomes

$$\frac{\Delta\rho}{\bar{\rho}} gH = (2\text{–}3)V^2 \quad (36)$$

or

$$V = \sqrt{\left[\frac{\Delta\rho}{\bar{\rho}} gH/(2\text{–}3)\right]}. \quad (37)$$

With heat transfer alone, and with practical conditions of $T_2 = 70°F$, $T_1 = 30°F$, $H = 1$ in and $L = 6$ in, for which the mean temperature is 50°F with $\alpha = 0{\cdot}78$ ft²/h and $\nu = 0{\cdot}56$ ft²/h, equation (23) can now be investigated.

With the given data and equation (37) the term VH/α has a value between 100 and 125; thus equation (23) reduces to the form of equation (11). Equation (30) becomes

$$\Delta T_{\text{avg}} \simeq 0{\cdot}98\,(T_2 - T_1). \quad (38)$$

It is now necessary to determine whether equation (36) can be expected to have the approximately correct form in the range of given conditions. Strictly speaking this cannot be done without knowing the distribution of flow in the opening. It seems reasonable to assume, however, that flow will occur somewhat as indicated in Fig. 1, in which case the Reynolds number, which indicates the range of validity of equation (36) for hydraulic flow in orifices, can be evaluated using about one-half of the opening width L as characteristic length. The Reynolds number $VL/2\nu$ so obtained has the value of about 400, which is sufficiently close to the range covered in orifice experiments to indicate the validity of equation (36).

Forming the Nusselt number using equation (37) yields the theoretical relation for air

$$Nu_H = \frac{1}{2\sqrt{(2\text{–}3)}} Gr_H{}^{1/2}Pr$$
$$= (0{\cdot}29\text{–}0{\cdot}35)Gr_H{}^{1/2}Pr. \qquad (39)$$

Experimental data for air would be expected to take the form of equation (13).

EXPERIMENTAL

Tests were carried out in the large wall-panel test unit used in previous tests on natural convection through openings in vertical partitions [2]. The apparatus [3] consists of two boxes 8 ft square and 4 ft deep (Fig. 2). One box (the warm side) is maintained at approximately 72°F by means of water circulated through the tubing of a panel on its inside wall. A separate tubing arrangement in the wall is separated from the panel by insulation and maintained automatically at the same temperature to prevent heat flow to the environment from the inner wall panel. The cold side can be maintained at any temperature down to about −20°F by means of a low temperature water–glycol–alcohol liquid from a separate cooling system that flows in the tubing of a wall panel and in a secondary finned tubing arrangement.

Since the test apparatus was designed for use with vertical walls it was necessary to build a special wall section (Fig. 2) in order to obtain a horizontal partition in which openings of various sizes could be cut for the tests. The test section was built in the form of a cubical box, 3 ft on a side, protruding from the wall.

By constructing all parts of the wall and test section of insulating material, a twofold advantage was afforded, (1) a large portion of the total heat transfer would occur across an opening, and (2) the convection conditions approximate closely those that would occur with density differences due to concentrations alone, the result being that the heat transfer test results would be expected to apply for mass transfer as well. The wall, with the exception of the top and bottom of the test section, was constructed of 2-in foamed polystyrene insulation on a ¼-in plywood backing. The partitions forming the top and bottom of the test section consisted solely of layers of foamed polystyrene.

Because of space limitations and for ready access to the test section the two boxes of the apparatus were separated by a distance of 2 ft. An insulated wall was then built around this region to assure a minimum load on the cooling system of the cold-side box.

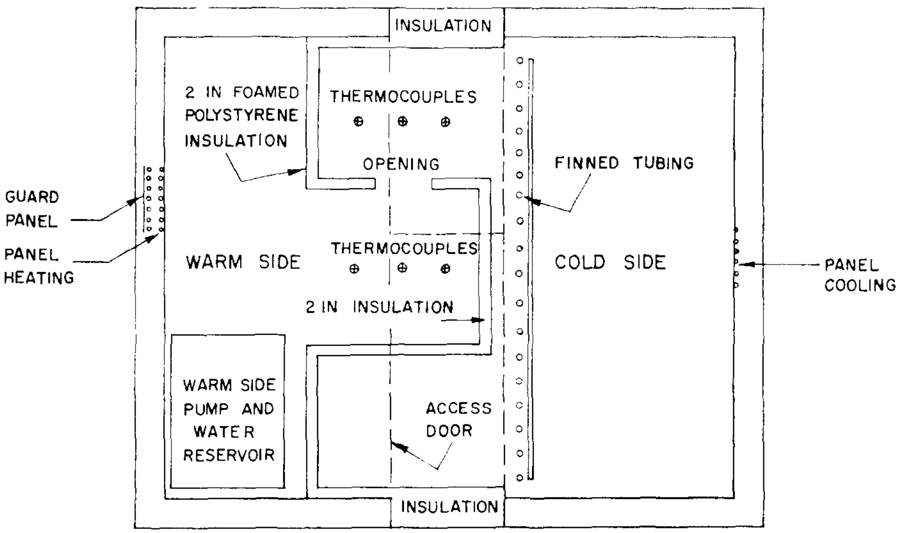

FIG. 2. Equipment arrangement for natural convection heat flow through an opening in a horizontal partition.

Instrumentation

Thirty-gauge copper–constantan thermocouples were arranged to measure the air temperature at five locations 10 in above the test partition and opening and also at five locations at the elevation of the centre of the box-like test section. Each set of thermocouples was arranged in the form of a square, 18 in on a side, with one thermocouple in the center directly above or below the center of a square opening in the test partition. Additional thermocouples were installed to measure the air temperature in the warm- and cold-side boxes at locations remote from the test section.

The temperature control for both warm and cold sides was sufficient to maintain the air temperature at any given location constant to within 0·2 degF. The total heat input to the warm side was obtained directly from continuous d.c. watt-meter recordings, the accuracy of the power input thus obtained being about 2 per cent.

Scope of tests and procedure

Tests were carried out with single square openings of nominal size: 6 × 6 in, 9 × 9 in and 12 × 12 in, with air temperature difference across the opening ranging from about 20 to 90 degF. The thickness of the partition of foamed polystyrene insulation was varied from 1 to 8 in. One set of tests was also made with a 12 × 12-in opening in an 8-in thick partition bevelled at a 45° angle to a thickness of 2 in. A few tests were also made with an opening in the lower partition of the test section. For this case a stable situation with no convection was to be expected.

Before carrying out tests with various openings it was necessary to calibrate the entire wall and test section with a blank partition of given thickness in place. This was done by determining the total heat transfer at various temperature differences between the air in the warm- and cold-side boxes. The results are given graphically in Fig. 3 where the heat flow in Btu/degF is plotted against air temperature difference. (The warm-side air temperature was maintained throughout at 72°F.)

With an opening in the partition, a small portion of the total heat transfer takes place by radiation, the amount of which was calculated by assuming that both the warm- and cold-side boxes behaved as black bodies, i.e.

$$\dot{q}_r = L^2\sigma(T_2^4 - T_1^4) \qquad (40)$$

where σ is the Stefan–Boltzmann constant and subscripts denote surface conditions. (Interchange with the edges of the opening was neglected.)

Test results

In accordance with equations (39) and (13) a relationship is to be expected between Nu_H/Pr and Gr_H, with the ratio of partition height to

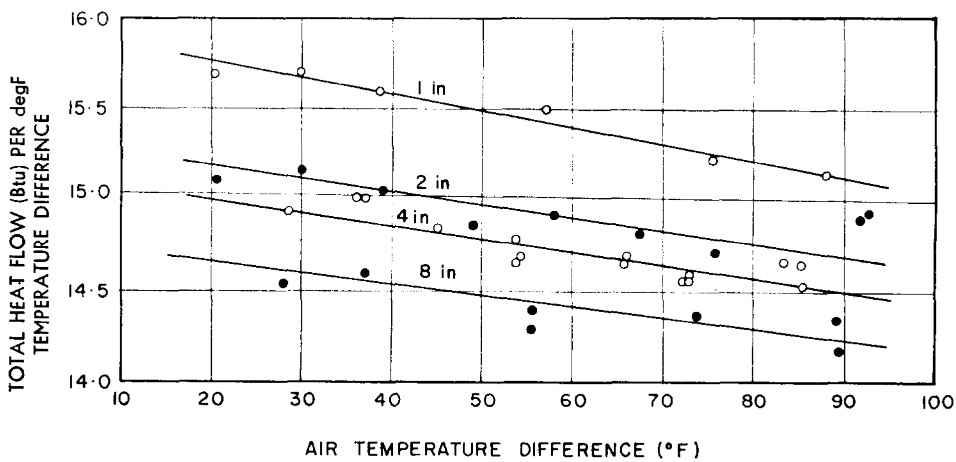

FIG. 3. Calibration of the test section and wall with horizontal partitions of various thicknesses.

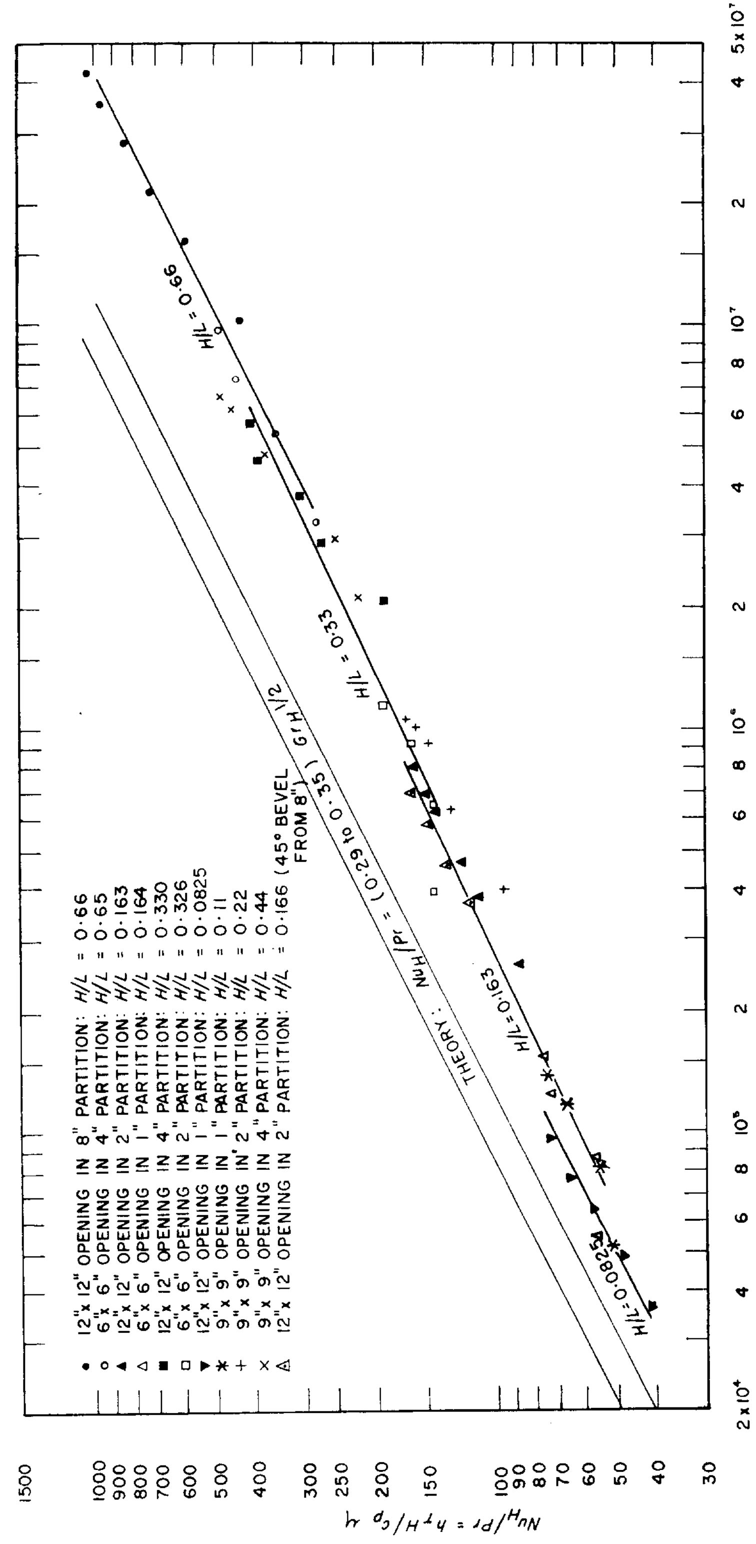

FIG. 4. Experimental results for the Nusselt number divided by the Prandtl number as a function of the Grashof number for different values of the ratio of partition thickness to opening width H/L.

opening width H/L as a secondary variable. These dimensionless groups were evaluated for all tests using the average of T_1 and T_2 to obtain air properties; the results are given in Fig. 4. (The air temperatures T_1 and T_2 employed in calculations are the averages of readings of the four outer thermocouples above and below the opening.) From Fig. 4 it will be noted that agreement with the ideal situation as represented by equation (39) is fairly good, but that the experimental results are all somewhat lower than called for by this equation. This effect, as might be expected, is most pronounced at high values of H/L because fluid friction was neglected in equation (39).

As may be seen from Fig. 4 the experimental accuracy of the data is not sufficient to determine the effect of the term H/L with great accuracy. The data were rearranged, however, in accordance with equation (13), assuming several values of the exponent on H/L in the term $Nu_H(H/L)^{\exp}/Pr$ and plotting the results against the Grashof number. A value of $\exp = \frac{1}{3}$ appeared to correlate the data with fair accuracy (Fig. 5). A mean curve through the points in this figure gives the equation:

$$Nu_H = 0{\cdot}0546\, Gr_H{}^{0{\cdot}55} Pr (L/H)^{1/3}. \quad (41)$$

From the discussion following equation (14) it will be appreciated that the exponents on Gr_H and (L/H) fall within the range expected from theoretical considerations. The range of validity of the equation is for

$$3 \times 10^4 < Gr_H < 4 \times 10^7$$

and

$$0{\cdot}0825 < H/L < 0{\cdot}66,$$

and should be correct for any value of the Prandtl number greater than that of air ($Pr = 0{\cdot}71$).

The few tests carried out with an opening in the lower partition of the test section indicated pure conduction heat flow through stratified air. The measured heat flow in this case was less than the experimental accuracy of measurement.

Extrapolation of the test results for low Prandtl or Schmidt numbers

Experimental data may be used to estimate the value of the Nusselt or Sherwood numbers for low values of the Prandtl or Schmidt numbers. Making use of equation (30) the "effective" Grashof number is:

$$Gr_e = Gr_H \left\{ \left[\frac{\exp(VH/a) + 1}{\exp(VH/a) - 1} \right] \frac{-2a}{VH} \right\} \quad (42)$$

where Gr_H is the Grashof number based on the temperature difference $(T_1 - T_2)$. Forming the dimensionless group VH/a with the help of equations (10), (13) and (42) leads to

$$\frac{VH}{a} = 2C_3(Gr_e)^{1/(2+bd)} \left(\frac{L}{H}\right)^{(ad-bd)/(2+bd)} Pr$$
$$= 2(Nu_H/Pr)_{\text{air}}\, Pr \quad (43)$$

where $(Nu_H/Pr)_{\text{air}}$ is the value of the Nusselt number divided by the Prandtl number obtained with air.

Inserting equation (10) into equation (23) yields for the Nusselt number

$$Nu_H = C_3 \left[\frac{\exp(VH/a) + 1}{\exp(VH/a) - 1} \right] (Gr_e)^{1/(2+bd)} \left(\frac{L}{H}\right)^{(ad-bd)/(2+bd)} Pr,$$

or

$$Nu_H(H/L)^{(ac-bd)/(2+bd)}\, Pr$$
$$= \left[\frac{\exp(VH/a) + 1}{\exp(VH/a) - 1} \right] \left[\frac{Nu_H(H/L)^{(ad-bd)/(2+bd)}}{Pr} \right]_{\text{air}}. \quad (44)$$

By inserting various values of $(Nu_H/Pr)_{\text{air}}$ corresponding to the range of tests into equation (43), values of VH/a are obtained which can be inserted into equation (42) to obtain Gr_H and into equation (44) to obtain $Nu_H(H/L)^{(ad-bd)/(2+bd)}/Pr$ for any given value of Pr. An example is given in Fig. 6 for $Pr = 0{\cdot}01$ as would be obtained with liquid sodium. (The solid portion of the curves corresponds to the range covered in the tests with air.) It will be noted that for $Pr = 0{\cdot}01$ and for a value of $Gr_H < 10^5$ practically all heat transfer is due to conduction even though fluid mixing and circulation still occurs.

DISCUSSION AND CONCLUSION

Equation (41) and Fig. 5, representing the relation obtained in heat-transfer tests with air as the fluid medium, should also be directly

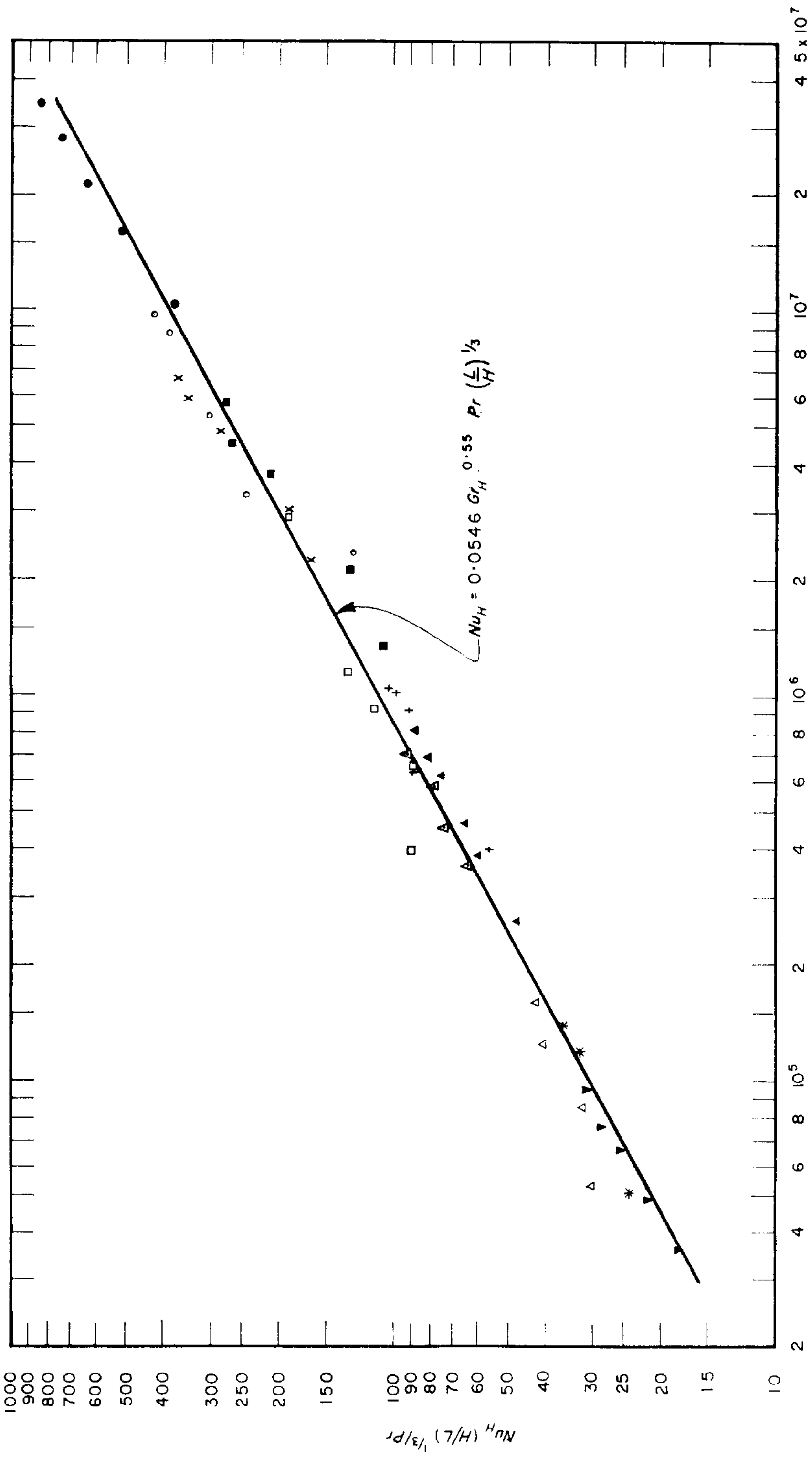

FIG. 5. Correlation of experimental results relating the Nusselt number, the Prandtl number, the Grashof number and the ratio H/L. (Designation of experimental points is the same as in Fig. 4.)

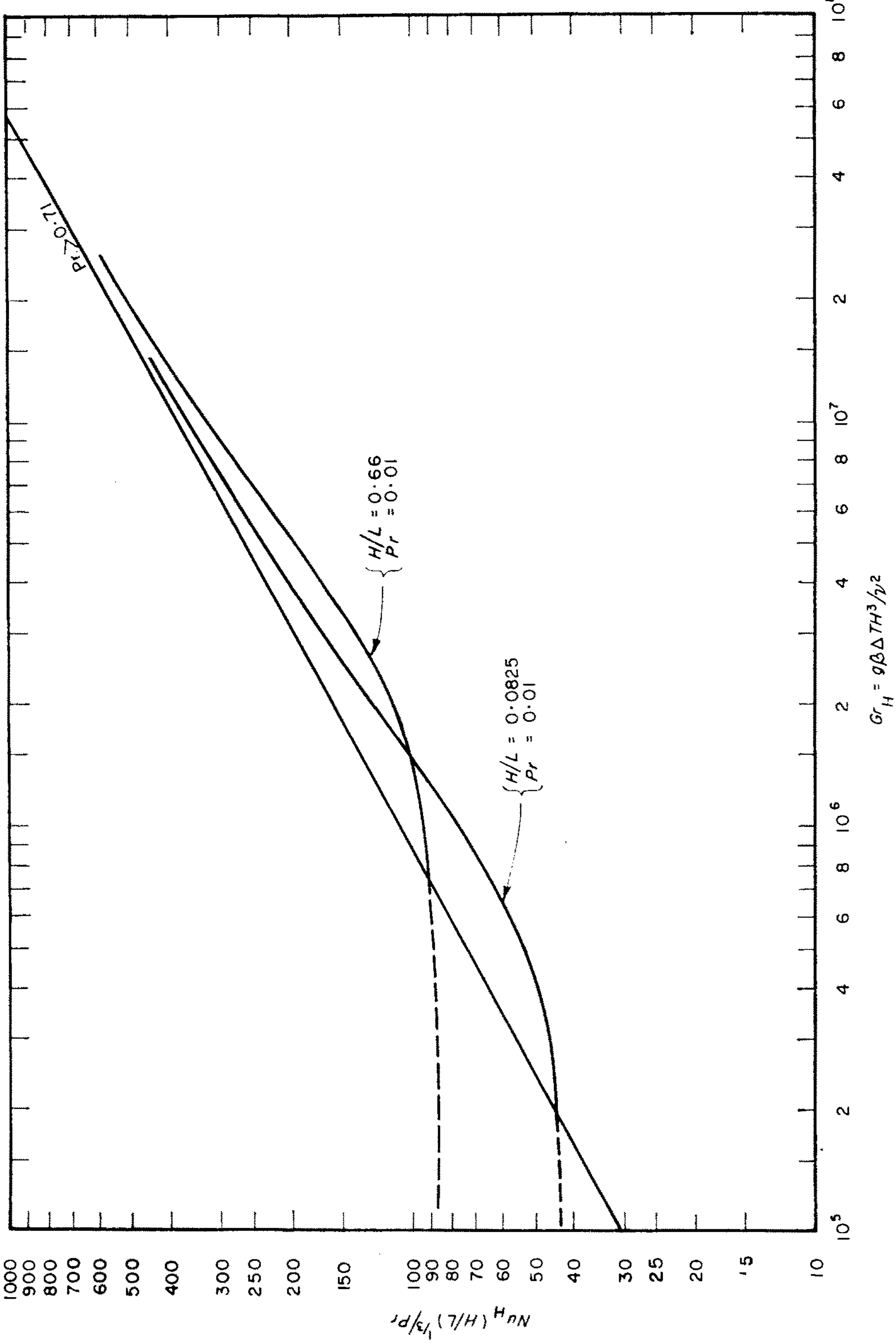

FIG. 6. Calculated values of $Nu_H(H/L)^{1/3}/Pr$ as a function of the Grashof number for a Prandtl number of 0·01 for two values of H/L.

applicable within the range of Gr_H and H/L covered in the tests for any value of Pr greater than about 0·1. The test results obtained may have been influenced to some extent by the configuration of the test section itself: of necessity it was relatively small compared with the openings due to space limitations within the test apparatus. So far as is known, however, no previous tests of this kind have been carried out and it is to be expected that the results are sufficiently accurate for practical purposes. The method devised for extending the data for fluids having low Prandtl or Schmidt numbers is not to be considered exact, owing principally to the inherent approximations involved in equations (15–31). For example, it was assumed that the temperatures on both sides of the partitions were everywhere constant, when in reality there is always a temperature gradient extending beyond the opening. Also, a gradient in temperature through the opening would be expected to have an additional influence on the velocity not included by defining the effective Grashof number as in equation (42). For practical purposes, however, the methods given here should be useful in estimating either heat or mass transfer for a wide range of conditions.

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Résumé—Cet article étudie la convection naturelle de l'air à travers des ouvertures carrées dans une paroi horizontale pour le cas du fluide le plus lourd au-dessus de la paroi. Les résultats d'essais donnant le nombre de Nusselt en fonction du nombre de Grashof et du rapport profondeur sur largeur des ouvertures concordent généralement bien avec la théorie. Bien que le nombre de Prandtl de l'air reste bien constant dans tous les essais, il a été néamoins possible, à l'aide de la théorie, de montrer approximativement l'influence de chaque nombre de Prandtl. Par suite de la grande résistance thermique de la paroi les résultats des mesures peuvent être directement appliqués au transport de masse. Les domaines suivants des variables ont été explorés au cours des essais: nombre de Grashof basé sur l'épaisseur de la paroi et la différence de température de l'air à travers l'ouverture $3 \times 10^4 < Gr_H < 4 \times 10^7$; rapport de l'épaisseur de la paroi à la section droite de l'ouverture $0{,}0825 < H/L < 0{,}66$ avec des ouvertures de 15 cm², 23 cm², 30 cm².

Zusammenfassung—Die natürliche Konvektion durch quadratische Öffnungen in einer waagerechten Trennwand mit dem schwereren Medium oben wurde für Luft als Konvektionsmedium untersucht. Die Ergebnisse liefern die Abhängigkeit der Nusselt-Zahl von der Grashof-Zahl und dem Verhältnis Öffnungsdicke zu Öffnungsweite und stimmen im allgemeinen mit der Theorie überein. Obwohl die Prandtl-Zahl der Luft für alle Versuche konstant blieb, war es doch möglich. mit Hilfe der Theorie, angenähert den zu erwartenden Einfluss anderer Prandtl-Zahlen zu bestimmen. Wegen des grossen thermischen Widerstands des Trennwandmaterials könnten die Ergebnisse auch direkt auf den Stoffübergang anzuwenden sein. Die Versuche umfassten folgenden Bereich von Variablen: Grashof-Zahl Gr_H auf die Trennwanddicke und die Differenz der Lufttemperaturen beiderseits der Öffnung bezogen $3 \times 10^4 < Gr_H < 4 \times 10^7$; Verhältnis der Trennwanddicke zu Breite der quadratischen Öffnung H/L, $0{,}0825 < H/L < 0{,}66$ bei Öffnungen von 152 × 152 mm; 228 × 228 mm und 305 × 305 mm.

Аннотация—Исследовался процесс естественной конвекции через квадратные отверстия в горизонтальной перегородке для случая наличия над перегородкой более тяжёлой жидкости. В качестве жидкой среды использовался воздух. Найдено, что экспериментальные результаты, устанавливающие зависимость между критериями Нуссельта, Грасгофа и отношением толщины отверстия к его ширине, в общем согласуются с требованиями теории. Значение критерия Прандтля для воздуха оставалось постоянным во всех опытах, тем не менее удалось теоретически показать возможное влияние для любых значений критерия Прандтля. Благодаря высокому термическому сопротивлению материала перегородки результаты опытов можно применить непосредственно к случаю переноса массы. В опытах использовался следующий диапазон значений переменных: критерий Грасгофа, Gr_H, в котором в качестве характерного размера взята толщина перегородки и разность температур воздуха поперёк отверстия, $3 \times 10^4 < Gr_H < 4 \times 10^7$. отношение толщины перегородки к стороне квадратного отверстия, H/L, $0{,}0825 < H/L < 0{,}66$ при отверстиях размером 6×6 дюймов, 9×9 дюймов и 12×12 дюймов.